In the previous section you learned how to use graphs to show the set of commodity bundles that a consumer can afford. In this section, you learn to put information about the consumer's preferences on the same kind of graph. Most of the problems ask you to draw indifference curves.

Sometimes we give you a formula for the indifference curve. Then all you have to do is graph a known equation. But in some problems, we give you only "qualitative" information about the consumer's preferences and ask you to sketch indifference curves that are consistent with this information. This requires a little more thought. Don't be surprised or disappointed if you cannot immediately see the answer when you look at a problem, and don't expect that you will find the answers hiding somewhere in your textbook. The best way we know to find answers is to "think and doodle." Draw some axes on scratch paper and label them, then mark a point on your graph and ask yourself, "What other points on the graph would the consumer find indifferent to this point?" If possible, draw a curve connecting such points, making sure that the shape of the line you draw reflects the features required by the problem. This gives you one indifference curve. Now pick another point that is preferred to the first one you drew and draw an indifference curve through it.

Jocasta loves to dance and hates housecleaning. She has strictly convex preferences. She prefers dancing to any other activity and never gets tired of dancing, but the more time she spends cleaning house, the less happy she is. Let us try to draw an indifference curve that is consistent with her preferences. There is not enough information here to tell us exactly where her indifference curves go, but there is enough information to determine some things about their shape. Take a piece of scratch paper and draw a pair of axes. Label the horizontal axis "Hours per day of housecleaning." Label the vertical axis "Hours per day of dancing." Mark a point a little ways up the vertical axis and write a 4 next to it. At this point, she spends 4 hours a day dancing and no time housecleaning. Other points that would be indifferent to this point would have to be points where she did more dancing *and* more housecleaning. The pain of the extra housekeeping should just compensate for the pleasure of the extra dancing. So an indifference curve for Jocasta must be upward sloping. Because she loves dancing and hates housecleaning, it must be that she prefers all the points above this indifference curve to all of the points on or below it. If Jocasta has strictly convex preferences, then it must be that if you draw a line between any two points on the same indifference curve, all the points on the line (except the endpoints) are preferred to the endpoints. For this to be the case, it must be that the indifference curve slopes upward ever more steeply as you move to the right along it. You should convince yourself of this by making some drawings on scratch paper. Draw an upward-sloping curve passing through the point $(0, 4)$

and getting steeper as one moves to the right.

When you have completed this workout, we hope that you will be able to do the following:

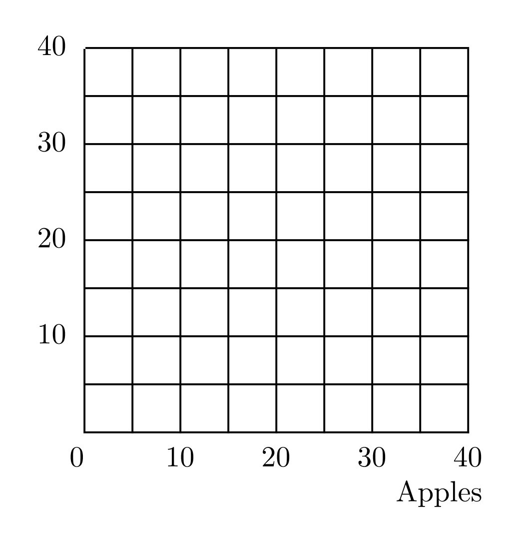
- Given the formula for an indifference curve, draw this curve, and find its slope at any point on the curve.
- Determine whether a consumer prefers one bundle to another or is indifferent between them, given specific indifference curves.
- Draw indifference curves for the special cases of perfect substitutes and perfect complements.
- Draw indifference curves for someone who dislikes one or both commodities.
- Draw indifference curves for someone who likes goods up to a point but who can get "too much" of one or more goods.
- Identify weakly preferred sets and determine whether these are convex sets and whether preferences are convex.
- Know what the marginal rate of substitution is and be able to determine whether an indifference curve exhibits "diminishing marginal rate of substitution."
- Determine whether a preference relation or any other relation between pairs of things is transitive, whether it is reflexive, and whether it is complete.

3.1 (0) Charlie likes both apples and bananas. He consumes nothing else. The consumption bundle where Charlie consumes x_A bushels of apples per year and x_B bushels of bananas per year is written as (x_A, x_B). Last year, Charlie consumed 20 bushels of apples and 5 bushels of bananas. It happens that the set of consumption bundles (x_A, x_B) such that Charlie is indifferent between (x_A, x_B) and $(20, 5)$ is the set of all bundles such that $x_B = 100/x_A$. The set of bundles (x_A, x_B) such that Charlie is just indifferent between (x_A, x_B) and the bundle $(10, 15)$ is the set of bundles such that $x_B = 150/x_A$.

(a) On the graph below, plot several points that lie on the indifference curve that passes through the point $(20, 5)$, and sketch this curve, using blue ink. Do the same, using red ink, for the indifference curve passing through the point $(10, 15)$.

(b) Use pencil to shade in the set of commodity bundles that Charlie weakly prefers to the bundle $(10, 15)$. Use blue ink to shade in the set of commodity bundles such that Charlie weakly prefers $(20, 5)$ to these bundles.

____



For each of the following statements about Charlie's preferences, write "true" or "false."

(c) $(30, 5) \sim (10, 15)$.______________________________.

(d) $(10, 15) \succ (20, 5)$.______________________________.

(e) $(20, 5) \succeq (10, 10)$.______________________________.

(f) $(24, 4) \succeq (11, 9.1)$.______________________________.

(g) $(11, 14) \succ (2, 49)$.______________________________.

(h) A set is convex if for any two points in the set, the line segment between them is also in the set. Is the set of bundles that Charlie weakly prefers to $(20, 5)$ a convex set?______________________________.

(i) Is the set of bundles that Charlie considers inferior to $(20, 5)$ a convex set?______________________________.

(j) The slope of Charlie's indifference curve through a point, (x_A, x_B), is known as his marginal __________ of ________________ at that point.

(k) Remember that Charlie's indifference curve through the point $(10, 10)$ has the equation $x_B = 100/x_A$. Those of you who know calculus will remember that the slope of a curve is just its derivative, which in this case is $-100/x_A^2$. (If you don't know calculus, you will have to take our word for this.) Find Charlie's marginal rate of substitution at the point,

$(10, 10)$.____.

(l) What is his marginal rate of substitution at the point $(5, 20)$?______

____.

(m) What is his marginal rate of substitution at the point $(20, 5)$?______

____.

(n) Do the indifference curves you have drawn for Charlie exhibit diminishing marginal rate of substitution?______________________________.

3.2 (0) Ambrose consumes only nuts and berries. Fortunately, he likes both goods. The consumption bundle where Ambrose consumes x_1 units of nuts per week and x_2 units of berries per week is written as (x_1, x_2). The set of consumption bundles (x_1, x_2) such that Ambrose is indifferent between (x_1, x_2) and $(1, 16)$ is the set of bundles such that $x_1 \geq 0$, $x_2 \geq 0$, and $x_2 = 20 - 4\sqrt{x_1}$. The set of bundles (x_1, x_2) such that $(x_1, x_2) \sim (36, 0)$ is the set of bundles such that $x_1 \geq 0$, $x_2 \geq 0$ and $x_2 = 24 - 4\sqrt{x_1}$.

(a) On the graph below, plot several points that lie on the indifference curve that passes through the point $(1, 16)$, and sketch this curve, using blue ink. Do the same, using red ink, for the indifference curve passing through the point $(36, 0)$.

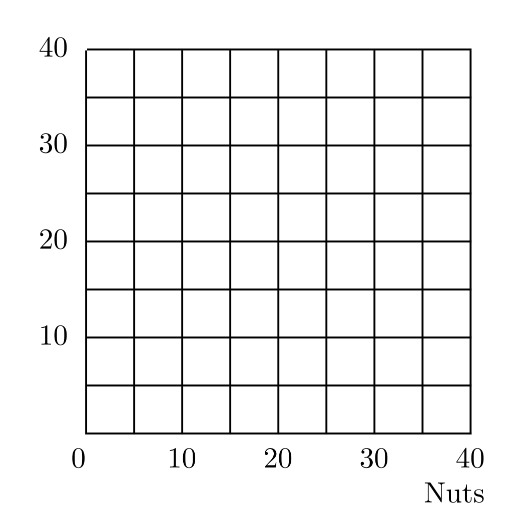
(b) Use pencil to shade in the set of commodity bundles that Ambrose weakly prefers to the bundle $(1, 16)$. Use red ink to shade in the set of all commodity bundles (x_1, x_2) such that Ambrose weakly prefers $(36, 0)$ to these bundles. Is the set of bundles that Ambrose prefers to $(1, 16)$ a convex set?____.

(c) What is the slope of Ambrose's indifference curve at the point $(9, 8)$? (Hint: Recall from calculus the way to calculate the slope of a curve. If you don't know calculus, you will have to draw your diagram carefully and estimate the slope.)_____________________________________.

____

(d) What is the slope of his indifference curve at the point $(4, 12)$? _______

____.



(e) What is the slope of his indifference curve at the point $(9, 12)$? _______

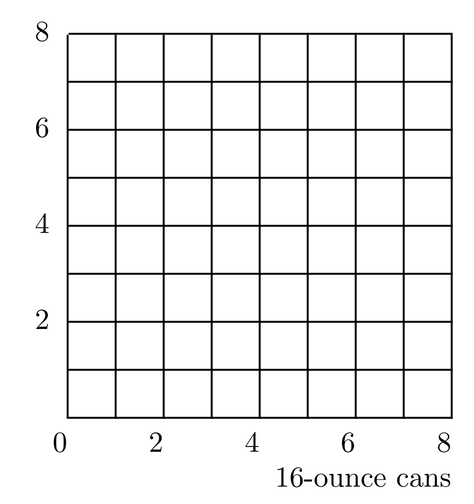
_______ at the point $(4, 16)$? _______________________.

(f) Do the indifference curves you have drawn for Ambrose exhibit diminishing marginal rate of substitution? _______________________.

(g) Does Ambrose have convex preferences? _______________________.

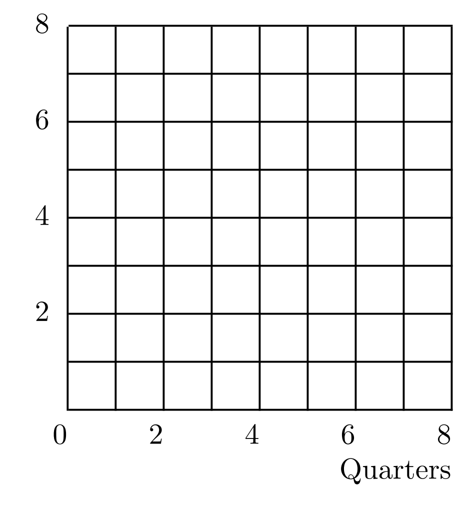
3.3 (0) Shirley Sixpack is in the habit of drinking beer each evening while watching "The Best of Bowlerama" on TV. She has a strong thumb and a big refrigerator, so she doesn't care about the size of the cans that beer comes in, she only cares about how much beer she has.

(a) On the graph below, draw some of Shirley's indifference curves between 16-ounce cans and 8-ounce cans of beer. Use blue ink to draw these indifference curves.



(b) Lorraine Quiche likes to have a beer while she watches "Masterpiece Theatre." She only allows herself an 8-ounce glass of beer at any one time. Since her cat doesn't like beer and she hates stale beer, if there is more than 8 ounces in the can she pours the excess into the sink. (She has no moral scruples about wasting beer.) On the graph above, use red ink to draw some of Lorraine's indifference curves.

3.4 (0) Elmo finds himself at a Coke machine on a hot and dusty Sunday. The Coke machine requires exact change—two quarters and a dime. No other combination of coins will make anything come out of the machine. No stores are open; no one is in sight. Elmo is so thirsty that the only thing he cares about is how many soft drinks he will be able to buy with the change in his pocket; the more he can buy, the better. While Elmo searches his pockets, your task is to draw some indifference curves that describe Elmo's preferences about what he finds.



_(a)_ If Elmo has 2 quarters and a dime in his pockets, he can buy 1 soft drink. How many soft drinks can he buy if he has 4 quarters and 2 dimes?

____.

_(b)_ Use red ink to shade in the area on the graph consisting of all combinations of quarters and dimes that Elmo thinks are just indifferent to having 2 quarters and 1 dime. (Imagine that it is possible for Elmo to have fractions of quarters or of dimes, but, of course, they would be useless in the machine.) Now use blue ink to shade in the area consisting of all combinations that Elmo thinks are just indifferent to having 4 quarters and 2 dimes. Notice that Elmo has indifference "bands," not indifference curves.

_(c)_ Does Elmo have convex preferences between dimes and quarters?

____.

_(d)_ Does Elmo always prefer more of both kinds of money to less? ______

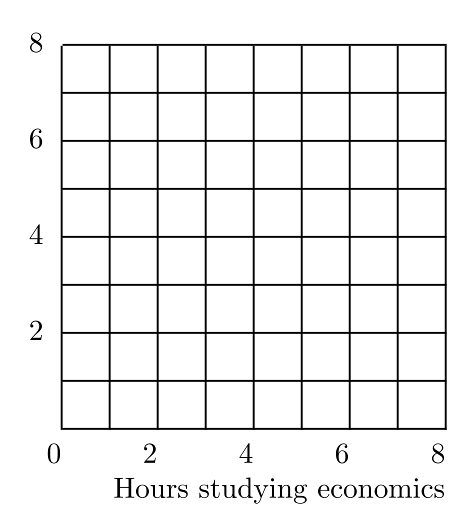
_(e)_ Does Elmo have a bliss point? ______________________.

_(f)_ If Elmo had arrived at the Coke machine on a Saturday, the drugstore across the street would have been open. This drugstore has a soda fountain that will sell you as much Coke as you want at a price of 4 cents an ounce. The salesperson will take any combination of dimes and quarters in payment. Suppose that Elmo plans to spend all of the money in his pocket on Coke at the drugstore on Saturday. On the graph above, use pencil or black ink to draw one or two of Elmo's indifference curves between quarters and dimes in his pocket. (For simplicity, draw your graph as if Elmo's fractional quarters and fractional dimes are accepted at the corresponding fraction of their value.) Describe these new indifference curves in words. ______________________________.

3.5 (0) Randy Ratpack hates studying both economics and history. The more time he spends studying either subject, the less happy he is. But Randy has strictly convex preferences.

_(a)_ Sketch an indifference curve for Randy where the two commodities are hours per week spent studying economics and hours per week spent studying history. Will the slope of an indifference curve be positive or negative? ______________________________.

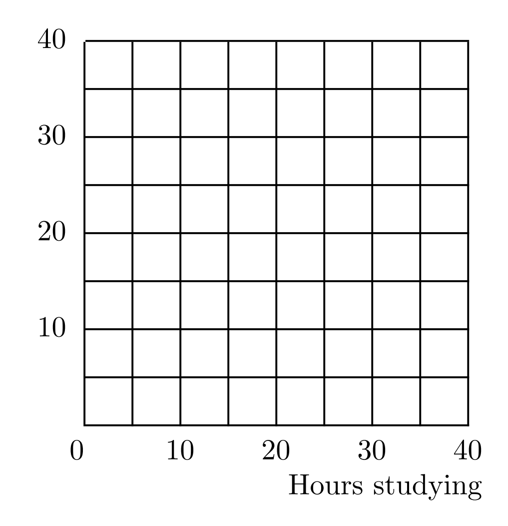
(b) Do Randy's indifference curves get steeper or flatter as you move from left to right along one of them?__________________________________.



3.6 (0) Flossy Toothsome likes to spend some time studying and some time dating. In fact her indifference curves between hours per week spent studying and hours per week spent dating are concentric circles around her favorite combination, which is 20 hours of studying and 15 hours of dating per week. The closer she is to her favorite combination, the happier she is.

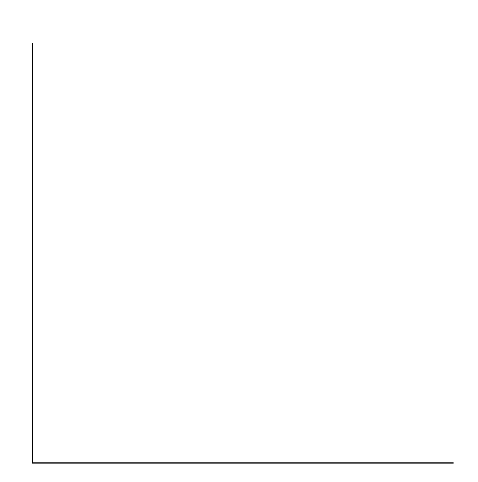
(a) Suppose that Flossy is currently studying 25 hours a week and dating 3 hours a week. Would she prefer to be studying 30 hours a week and dating 8 hours a week? __________ (Hint: Remember the formula for the distance between two points in the plane?)

(b) On the axes below, draw a few of Flossy's indifference curves and use your diagram to illustrate which of the two time allocations discussed above Flossy would prefer.



3.7 (0) Joan likes chocolate cake and ice cream, but after 10 slices of cake, she gets tired of cake, and eating more cake makes her less happy. Joan always prefers more ice cream to less. Joan's parents require her to eat everything put on her plate. In the axes below, use blue ink to draw a set of indifference curves that depict her preferences between plates with different amounts of cake and ice cream. Be sure to label the axes.

(a) Suppose that Joan's preferences are as before, but that her parents allow her to leave anything on her plate that she doesn't want. On the graph below, use red ink to draw some indifference curves depicting her preferences between plates with different amounts of cake and ice cream.

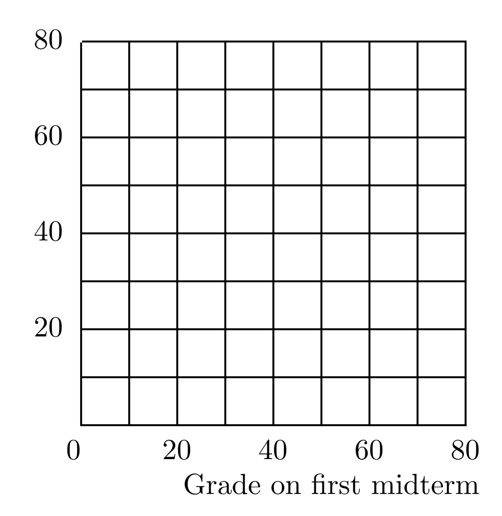


3.8 (0) Professor Goodheart always gives two midterms in his communications class. He only uses the higher of the two scores that a student gets on the midterms when he calculates the course grade.

(a) Nancy Lerner wants to maximize her grade in this course. Let x_1 be her score on the first midterm and x_2 be her score on the second midterm. Which combination of scores would Nancy prefer, $x_1 = 20$ and $x_2 = 70$ or $x_1 = 60$ and $x_2 = 60$?________________________.

(b) On the graph below, use red ink to draw an indifference curve showing all of the combinations of scores that Nancy likes exactly as much as $x_1 = 20$ and $x_2 = 70$. Also use red ink to draw an indifference curve showing the combinations that Nancy likes exactly as much as $x_1 = 60$ and $x_2 = 60$.

(c) Does Nancy have convex preferences over these combinations?______.



(d) Nancy is also taking a course in economics from Professor Stern. Professor Stern gives two midterms. Instead of discarding the lower grade, Professor Stern discards the higher one. Let x_1 be her score on the first midterm and x_2 be her score on the second midterm. Which combination of scores would Nancy prefer, $x_1 = 20$ and $x_2 = 70$ or $x_1 = 60$ and $x_2 = 50$?________________________.

(e) On the graph above, use blue ink to draw an indifference curve showing all of the combinations of scores on her econ exams that Nancy likes exactly as well as $x_1 = 20$ and $x_2 = 70$. Also use blue ink to draw an indifference curve showing the combinations that Nancy likes exactly as well as $x_1 = 60$ and $x_2 = 50$. Does Nancy have convex preferences over these combinations?________________________.

3.9 (0) Mary Granola loves to consume two goods, grapefruits and avocados.

(a) On the graph below, the slope of an indifference curve through any point where she has more grapefruits than avocados is -2. This means that when she has more grapefruits than avocados, she is willing to give

up ________ grapefruit(s) to get one avocado.

(b) On the same graph, the slope of an indifference curve at points where she has fewer grapefruits than avocados is $-1/2$. This means that when she has fewer grapefruits than avocados, she is just willing to give up

_______ grapefruit(s) to get one avocado.

(c) On this graph, draw an indifference curve for Mary through bundle $(10A, 10G)$. Draw another indifference curve through $(20A, 20G)$.

Grapefruits

40

30

20

10

0 10 20 30 40

Avocados

(d) Does Mary have convex preferences?________________________.

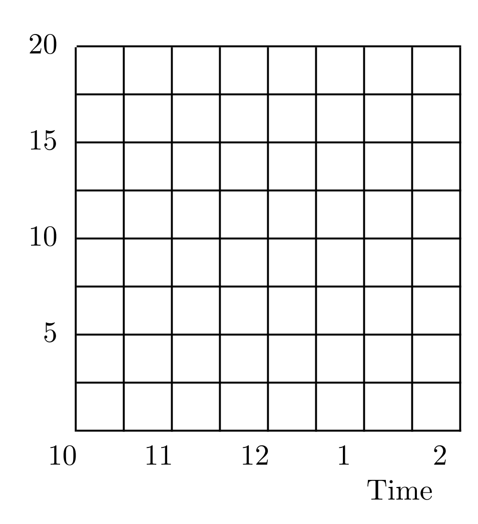
3.10 (2) Ralph Rigid likes to eat lunch at 12 noon. However, he also likes to save money so he can buy other consumption goods by attending the "early bird specials" and "late lunchers" promoted by his local diner. Ralph has 15 dollars a day to spend on lunch and other stuff. Lunch at noon costs \$5. If he delays his lunch until t hours after noon, he is able to buy his lunch for a price of $\$5-t$. Similarly if he eats his lunch t hours before noon, he can buy it for a price of $\$5-t$. (This is true for fractions of hours as well as integer numbers of hours.)

(a) If Ralph eats lunch at noon, how much money does he have per day

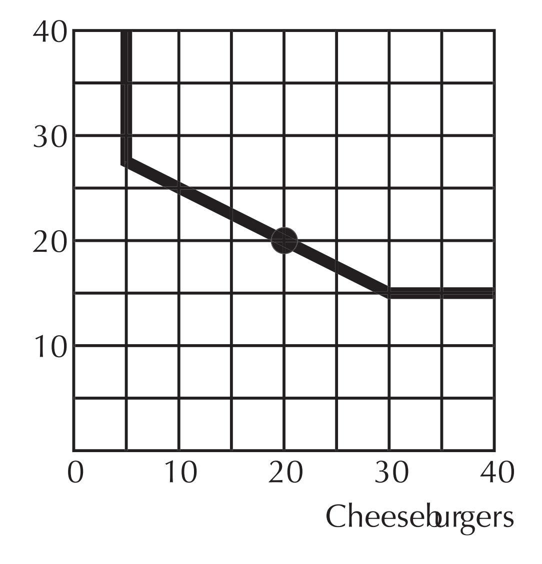
to spend on other stuff?________________________________.

(b) How much money per day would he have left for other stuff if he ate at 2 P.M.?______________________________.

(c) On the graph below, use blue ink to draw the broken line that shows combinations of meal time and money for other stuff that Ralph can just afford. On this same graph, draw some indifference curves that would be consistent with Ralph choosing to eat his lunch at 11 A.M.



3.11 (0) Henry Hanover is currently consuming 20 cheeseburgers and 20 Cherry Cokes a week. A typical indifference curve for Henry is depicted below.





(a) If someone offered to trade Henry one extra cheeseburger for every Coke he gave up, would Henry want to do this?____________________.

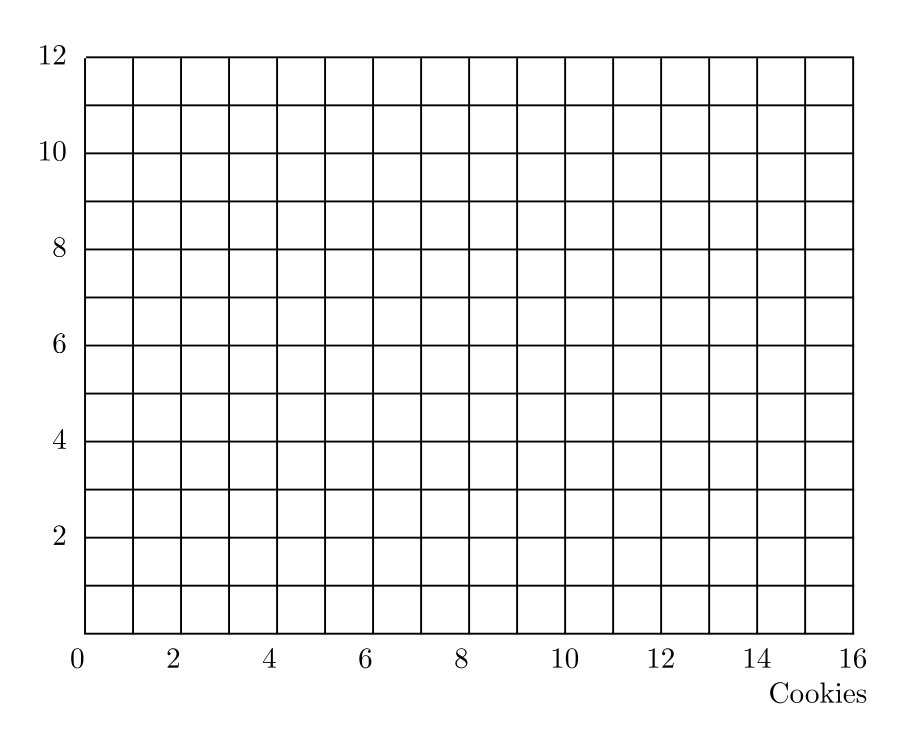
(b) What if it were the other way around: for every cheeseburger Henry gave up, he would get an extra Coke. Would he accept this offer?________

____.

(c) At what rate of exchange would Henry be willing to stay put at his current consumption level?____________________________________.

3.12 (1) Tommy Twit is happiest when he has 8 cookies and 4 glasses of milk per day. Whenever he has more than his favorite amount of either food, giving him still more makes him worse off. Whenever he has less than his favorite amount of either food, giving him more makes him better off. His mother makes him drink 7 glasses of milk and only allows him 2 cookies per day. One day when his mother was gone, Tommy's sadistic sister made him eat 13 cookies and only gave him 1 glass of milk, despite the fact that Tommy complained bitterly about the last 5 cookies that she made him eat and begged for more milk. Although Tommy complained later to his mother, he had to admit that he liked the diet that his sister forced on him better than what his mother demanded.

(a) Use black ink to draw some indifference curves for Tommy that are consistent with this story.



(b) Tommy's mother believes that the optimal amount for him to consume is 7 glasses of milk and 2 cookies. She measures deviations by absolute values. If Tommy consumes some other bundle, say, (c, m), she measures his departure from the optimal bundle by $D = |7 - m| + |2 - c|$. The larger D is, the worse off she thinks Tommy is. Use blue ink in the graph above to sketch a few of Mrs. Twit's indifference curves for Tommy's consumption. (Hint: Before you try to draw Mrs. Twit's indifference curves, we suggest that you take a piece of scrap paper and draw a graph of the locus of points (x_1, x_2) such that $|x_1| + |x_2| = 1$.)

3.13 (0) Coach Steroid likes his players to be big, fast, and obedient. If player A is better than player B in two of these three characteristics, then Coach Steroid prefers A to B, but if B is better than A in two of these three characteristics, then Steroid prefers B to A. Otherwise, Steroid is indifferent between them. Wilbur Westinghouse weighs 340 pounds, runs very slowly, and is fairly obedient. Harold Hotpoint weighs 240 pounds, runs very fast, and is very disobedient. Jerry Jacuzzi weighs 150 pounds, runs at average speed, and is extremely obedient.

(a) Does Steroid prefer Westinghouse to Hotpoint or vice versa?______

____.

(b) Does Steroid prefer Hotpoint to Jacuzzi or vice versa?____________

____.

(c) Does Steroid prefer Westinghouse to Jacuzzi or vice versa?_________

____.

(d) Does Coach Steroid have transitive preferences?__________________.

(e) After several losing seasons, Coach Steroid decides to change his way of judging players. According to his new preferences, Steroid prefers player A to player B if player A is better in all three of the characteristics that Steroid values, and he prefers B to A if player B is better at all three things. He is indifferent between A and B if they weigh the same, are equally fast, and are equally obedient. In all other cases, Coach Steroid simply says "A and B are not comparable."

(f) Are Coach Steroid's new preferences complete?__________________.

____

(g) Are Coach Steroid's new preferences transitive?_________________.

(h) Are Coach Steroid's new preferences reflexive?_________________.

3.14 (0) The Bear family is trying to decide what to have for dinner. Baby Bear says that his ranking of the possibilities is (honey, grubs, Goldilocks). Mama Bear ranks the choices (grubs, Goldilocks, honey), while Papa Bear's ranking is (Goldilocks, honey, grubs). They decide to take each pair of alternatives and let a majority vote determine the family rankings.

(a) Papa suggests that they first consider honey vs. grubs, and then the winner of that contest vs. Goldilocks. Which alternative will be chosen?

____.

(b) Mama suggests instead that they consider honey vs. Goldilocks and then the winner vs. grubs. Which gets chosen?_________________.

(c) What order should Baby Bear suggest if he wants to get his favorite food for dinner?_________________________________.

(d) Are the Bear family's "collective preferences," as determined by voting, transitive?_________________________________.

3.15 (0) Olson likes strong coffee, the stronger the better. But he can't distinguish small differences. Over the years, Mrs. Olson has discovered that if she changes the amount of coffee by more than one teaspoon in her six-cup pot, Olson can tell that she did it. But he cannot distinguish differences smaller than one teaspoon per pot. Where A and B are two different cups of coffee, let us write $A \succ B$ if Olson prefers cup A to cup B. Let us write $A \succeq B$ if Olson either prefers A to B, or can't tell the difference between them. Let us write $A \sim B$ if Olson can't tell the difference between cups A and B. Suppose that Olson is offered cups A, B, and C all brewed in the Olsons' six-cup pot. Cup A was brewed using 14 teaspoons of coffee in the pot. Cup B was brewed using 14.75 teaspoons of coffee in the pot and cup C was brewed using 15.5 teaspoons of coffee in the pot. For each of the following expressions determine whether it is true of false.

(a) $A \sim B$.___________________________________.

(b) $B \sim A$.___________________________________.

(c) $B \sim C$.____.

(d) $A \sim C$.____.

(e) $C \sim A$.____.

(f) $A \succeq B$.____.

(g) $B \succeq A$.____.

(h) $B \succeq C$.____.

(i) $A \succeq C$.____.

(j) $C \succeq A$.____.

(k) $A \succ B$.____.

(l) $B \succ A$.____.

(m) $B \succ C$.____.

(n) $A \succ C$.____.

(o) $C \succ A$.____.

(p) Is Olson's "at-least-as-good-as" relation, $\succeq$, transitive?________________.

(q) Is Olson's "can't-tell-the-difference" relation, $\sim$, transitive?__________.

(r) is Olson's "better-than" relation, $\succ$, transitive.________________________.